① $\int \frac{(\ln x)^4}{x} \cdot dx$

$\boxed{y = \ln x} \to dy = \frac{1}{x} \cdot dx$

$\to \boxed{x \cdot dy = dx}$

$\int \frac{y^4}{x} \cdot x \cdot dy = \int y^4 \cdot dy$

$= \frac{y^5}{5} + C = \frac{(\ln x)^5}{5} + C$ ⓓ

② $\int (1-2x)\sqrt[3]{x^2-x} \cdot dx$

$\boxed{y = \sqrt[3]{x^2-x}} \to y^3 = x^2 - x$

$\to 3y^2\, dy = (2x-1) \cdot dx$

$\to \boxed{\frac{3y^2\, dy}{2x-1} = dx}$

$\int \overset{-1}{\cancel{(1-2x)}} \cdot y \cdot \frac{3y^2 \cdot dy}{\cancel{2x-1}}$

$= \int -3y^3 \cdot dy = \frac{-3\, y^4}{4} + C$

$= \frac{-3}{4} \sqrt[3]{(x^2-x)^4} + C$ ⓒ

③ $p'(x) = \frac{-6x}{\sqrt{8+x^2}}$

عدد القطع المبيعة بالمئات ← 100 قطعة

١ سعرها 10JD

شرط أولي $\to \boxed{P(1) = 10}$

$\to P(x) = \int p'(x) \cdot dx$

$= \int \frac{-6x}{\sqrt{8+x^2}} \cdot dx \qquad \boxed{y = \sqrt{8+x^2}}$

$\to y^2 = 8 + x^2 \to 2y\,dy = 2x \cdot dx$

$\frac{2y \cdot dy}{2x} = dx \to \boxed{dx = \frac{y}{x} \cdot dy}$

$P(x) = \int \frac{-6\cancel{x}}{\cancel{y}} \cdot \frac{\cancel{y}}{\cancel{x}} \cdot dy$

$= \int -6 \cdot dy = -6y + C$

$\boxed{P(x) = -6\sqrt{8+x^2} + C}$

$P(1) = -6\sqrt{9} + C = 10$

$-18 + C = 10 \to \boxed{C = 28}$

$P(x) = -6\sqrt{8+x^2} + 28$ ⓒ

④ $\int x \cdot \sin(x^2+1) \cdot dx$

$\boxed{y = x^2+1} \to dy = 2x \cdot dx$

$\dfrac{dy}{2x} = dx$

$= \int \cancel{x} \cdot \sin(y) \cdot \dfrac{dy}{2\cancel{x}}$

$= \int \frac{1}{2} \sin y \cdot dy = -\frac{1}{2} \cos y + c$

$= -\frac{1}{2} \cos(x^2+1) + c$ ⓑ

⑤ $\int x (4)^{x^2} \cdot dx$

$\boxed{y = x^2} \to dy = 2x \cdot dx \to \dfrac{dy}{2x} = dx$

$\int x (4)^y \cdot \dfrac{dy}{2x} = \int \frac{1}{2} (4)^y \cdot dy$

$= \frac{1}{2} \dfrac{(4)^y}{\ln 4} + c$

$= \dfrac{(4)^y}{2 \ln 4} + c = \dfrac{(4)^{x^2}}{2 \ln 4} + c$ ⓑ

⑥ $\int \sin^3 x \cdot dx$

$= \int \sin x \cdot \sin^2 x \cdot dx$

$= \int \sin x \cdot (1 - \cos^2 x) \cdot dx$

$\boxed{y = \cos x} \to dy = -\sin x \cdot dx$

$\dfrac{dy}{-\sin x} = dx$

$= \int \cancel{\sin x} (1 - y^2) \cdot \dfrac{dy}{-\cancel{\sin x}}$

$= \int -(1 - y^2) \cdot dy = \int (y^2 - 1) \cdot dy$

$= \dfrac{y^3}{3} - y + c$

$= \dfrac{\cos^3 x}{3} - \cos x + c$

$= \frac{1}{3} \cos^3 x - \cos x + c$ ⓒ

⑦ $\int_0^{\frac{\pi}{4}} \tan x \sec^3 x \cdot dx$

$= \int_0^{\frac{\pi}{4}} \tan x \sec x \cdot \sec^2 x \cdot dx$

$\boxed{y = \sec x} \to dy = \sec x \tan x \cdot dx$

$\frac{dy}{\sec x \tan x} = dx$

$x = 0 \to y = \sec 0 = 1$

$x = \frac{\pi}{4} \to y = \sec\frac{\pi}{4} = \sqrt{2}$

$\Rightarrow \int_1^{\sqrt{2}} \cancel{\tan x}\,\cancel{\sec x} \cdot y^2 \cdot \frac{dy}{\cancel{\sec x}\,\cancel{\tan x}}$

$= \int_1^{\sqrt{2}} y^2 \cdot dy = \frac{y^3}{3}\Big|_1^{\sqrt{2}}$

$= \frac{(\sqrt{2})^3}{3} - \frac{(1)^3}{3} = \frac{\sqrt{2}\sqrt{2}\sqrt{2}}{3} - \frac{1}{3}$

$= \frac{2\sqrt{2}}{3} - \frac{1}{3} = \frac{1}{3}(2\sqrt{2} - 1)$

ⓒ

⑧ $\int \cot^3 x \cdot dx = \int \cot x \cdot \cot^2 x \cdot dx$

$= \int \cot x (\csc^2 x - 1) \cdot dx$

$= \int (\cot x \csc^2 x - \cot x)\, dx$

$= \boxed{\underbrace{\int \cot x \csc^2 x \cdot dx}_{*} - \underbrace{\int \cot x \cdot dx}_{**}}$

(*) $\int \cot x \csc^2 x \cdot dx$

$\boxed{y = \cot x} \to dy = -\csc^2 x \cdot dx$

$\frac{dy}{-\csc^2 x} = dx$

$\int y \cdot \cancel{\csc^2 x} \cdot \frac{dy}{-\cancel{\csc^2 x}} = \int -y \cdot dy$

$= -\frac{y^2}{2} + c = \boxed{-\frac{\cot^2 x}{2} + c}$

(**) $\int \cot x \cdot dx = \int \frac{\cos x}{\sin x} \cdot dx$

مشتقة المقام $= \cos x =$ البسط

$\to \int \cot x \cdot dx = \boxed{\ln|\sin x| + c}$

$\to \int \cot^3 x \cdot dx = \frac{-\cot^2 x}{2} - \ln|\sin x| + c$

$= -\left(\frac{\cot^2 x}{2} + \ln|\sin x|\right) + c$